MATH 1910, Fall 2016
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Sections: 201, 202 **Name:**________________________

Chapter 5 Review

Fundamental Identities

- $$\sin^2 x + \cos^2 x = \underline{\hspace{5cm}}$$
- $$1 + \tan^2 x = \underline{\hspace{5cm}}$$
- $$1 + \cot^2 x = \underline{\hspace{5cm}}$$

Important Integration Formulas

- $$\int u^n \ du \ = \underline{\hspace{4cm}} (n \neq 1)$$
- $$\int \sin u \ du = \underline{\hspace{5cm}}$$
- $$\int \cos u \ du = \underline{\hspace{5cm}}$$
- $$\int \sec^2 u \ du = \underline{\hspace{5cm}}$$
- $$\int \csc^2 u \ du = \underline{\hspace{5cm}}$$
- $$\int \sec u \tan u \ du = \underline{\hspace{4cm}}$$
- $$\int \csc u \cot u \ du = \underline{\hspace{4cm}}$$

Important Theorems and Definitions

- State the Comparison Theorem:
- F is called an *antiderivative* of f if ______________
- State the Fundamental Theorem of Calculus, Part I
- (Fundamental Theorem of Calculus, Part II) Assume that f is continuous on an open interval I and let a be a point in I. Then the area function, $A(x)$, with lower limit a is ______________ and $A'(x)$ is ______________, or equivalently,

$$\frac{d}{dx}\int_a^x f(t)dt = \underline{\hspace{4cm}}.$$

- Consider the function

$$G(x) = \int_a^{g(x)} f(t)dt.$$

Find $G'(x)$ ______________.

- The net change in a quantity $s(t)$ is equal to the integral of its rate of change:

$$s(t_2) - s(t_1) = \underline{\hspace{3cm}}.$$

- For an object traveling in a straight line at velocity $v(t)$, we have

  displacement during $[t_1, t_2] = \underline{\hspace{3cm}}$ and,

  total distance traveled during $[t_1, t_2] = \underline{\hspace{3cm}}$.

Express the limit as an integral (or multiple of an integral) and evaluate.

1.
$$\lim_{N\to\infty} \frac{\pi}{6N} \sum_{j=1}^{N} \sin\left(\frac{\pi}{3} + \frac{\pi j}{6N}\right)$$

2.
$$\lim_{N\to\infty} \frac{3}{N} \sum_{k=0}^{N-1} \left(10 + \frac{3k}{N}\right)$$

Calculate the definite or indefinite integral.

1.
$$\int (y+2)^4 \, dy$$

2.
$$\int (9t^{-2/3} + 4t^{7/3}) \, dt$$

3.
$$\int_{-2}^{4} |(x-1)||x-3| \, dx$$

4.
$$\int t^2\sqrt{t+8} \, dt$$

5.
$$\int \frac{\sec^2 t}{(\tan t - 1)^2} \, dt$$

6.
$$\int_0^{\pi/2} \sec^2(\cos\theta)\sin\theta \, d\theta$$

7.
$$\int \csc^2(9-2\theta) \, d\theta$$

8.
$$\int \frac{(x^2+1)}{(x^3+3x)^4} \, dx$$

9.
$$\int_0^{\pi/6} \sin x \cos^4 x \, dx$$

Solve the differential equation with the given initial condition.

1.
$$\frac{dy}{dx} = \sec^2 x, y(\pi/4) = 2$$

2. Find $f(t)$ if $f''(t) = 1 - 2t$, $f(0) = 2$, and $f'(0) = -1$.

3. At time $t = 0$, a driver begins decelerating at a constant rate of $= -10m/s^2$ and comes to a halt after traveling $500m$. Find the velocity at $t = 0$.

Additional Problems

1. Find the local minima, the local maxima, and the inflection points of

$$A(x) = \int_3^x \frac{t\ dt}{t^2+1}$$

2. On a typical day, a city consumes water at the rate of $r(t) = 100 + 72t - 3t^2$ (in thousands of gallons per hour), where t is the number of hours past midnight. What is the daily water consumption? How much water is consumed between 6 PM and midnight?

3. Evaluate the integral below, using the properties of odd functions.

$$\int_{-8}^{8} \frac{x^{15}}{3+\cos^2 x}\ dx.$$

4. Find the following:

   $G'(x)$, where

$$G(x) = \int_{-2}^{\sin x} t^3\ dt.$$

   $G'(x)$ and $G'(2)$ where,

$$G(x) = \int_0^{x^3} \sqrt{t+1}\ dt.$$

5. Use the comparison theorem to prove that

$$2 \le \int_1^2 2^x\ dx \le 4.$$